PART I

A. Find the **exact values** of the following without a calculator:

1. $\sin 15°$
2. $\cos\frac{\pi}{8}$
3. $1-2\sin^2 15°$
4. $\tan(75°)$

B. Simplify each of the following:

1. $2\sin(x+20°)\cos(x+20°)$
2. $1-2\sin^2\left(\frac{1}{4}\pi\right)$
3. $\left(\sin x-\cos x\right)^2$

C. Find the values of sin $2x$, cos $2x$, and tan $2x$ in each of the following *without* a calculator.

1. $\sin x=\frac{3}{5}$
2. $\cos x=\frac{12}{13}$
3. $\tan x=-\frac{3}{4}$

D. If $3\sin 2x=2\cos x$, find the values of cos $2x$.

E. If $\cos x=-\frac{7}{25}$, find $\sin\frac{1}{2}x$, $\cos\frac{1}{2}x$, and $\tan\frac{1}{2}x$ *without* a calculator.

F. Simplify $\frac{\cos 2x+1}{\cos x}$

PART II

A. Solve the following trigonometric equations (no calculators). If no domain is given, give the *general* solution (all solutions). If the domain is given in degrees, give the answers in degrees. If the domain is given in radians, give the answers in radians.

1. $2\cos x = 1$
2. $\tan x + 1 = 0,\ 0 \le x \le 360°$
3. $4\sin x + 3 = 0,\ -\pi \le x \le \pi$
4. $2\cos^2 x + 3\cos x - 2 = 0,\ 0 \le x \le 2\pi$
5. $2\cos^2 x - \sin x = 1, -180° \le x \le 180$
6. $\sqrt{3}\cos x - \sin x = 0,\ -\pi \le x \le \pi$
7. $\cos 2x - \cos x = 0,\ 0 \le x \le 360°$
8. $2\sin(x - 60°) = 1,\ 0 \le x \le 360°$
9. $3\cos\left(x - \frac{1}{6}\pi\right) + 1 = 0,\ -\pi \le x \le \pi$
10. $3\sin 3x + 1 = 0,\ 0 \le x \le \pi$
11. $\tan 4x = \sqrt{3},\ 0 \le x \le 180°$
12. $\frac{1}{\tan x} + \tan x = 4,\ -\pi \le x \le \pi$

B. Simplify the following:

1. $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$
2. $\cos^2 1\frac{1}{2} + \sin^2 1\frac{1}{2}$
3. $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$

C. Prove the following whenever both sides have meaning:

1. $2\cot^2 x - 3\csc^2 x = -2 - \csc^2 x = -3 - \cot^2 x$
2. $\sin A \sec A \cot A + \cos A \csc A \tan A = 2$
3. $\cot^2 A \sec^2 A + \tan^2 A \csc^2 A = \sec^2 A + \csc^2 A$
4. $2\tan^2 A - 3\sec^2 A = 1 - 4\sec^2 A + 3\tan^2 A$
5. $(\cos A - \sin A + 1)^2 = 2(1 + \cos A)(1 - \sin A)$
6. $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = 2\csc x$
7. $\dfrac{2\cos A}{\sin A - \cos A} + 1 = \dfrac{\tan A + 1}{\tan A - 1}$
8. $\dfrac{1 + \sec A}{1 - \sec A} = -(\cot A + \csc A)^2$
9. $\dfrac{(1 - \sin A)(1 - \cot A)}{(1 - \csc A)(1 - \tan A)} = \cos A$

D. Expand, and/or simplify where possible. One step is enough in cases where an expansion would be extremely long (such as #1).

1. $\cos 3(A + B)$
2. $\sin\left(\dfrac{\pi}{3} + 2A\right)$
3. $\dfrac{\tan\frac{1}{4}\pi + \tan A}{1 - \tan\frac{1}{4}\pi \tan A}$
4. $\dfrac{\tan 80° - \tan 35°}{\tan 80° + \cot 35°}$

E. Express as a single trigonometric function:

1. $\cos 55° \cos 35° - \sin 55° \sin 35°$
2. $\dfrac{1}{2}\sin A - \dfrac{1}{2}\sqrt{3}\cos A$

F. If A is an angle in quadrant 2 and B is an angle in quadrant 3 such that $\sin A = \dfrac{4}{5}$ and $\tan B = \dfrac{8}{15}$, evaluate $\cos(A - B)$, $\sin(A + B)$, and $\cos(A + B)$ without a calculator.

G. Prove, whenever both expressions have meaning:

1. $\sin(a+b)+\sin(a-b)=2\sin a\cos b$
2. $\sin B+\cos A\sin(A-B)=\sin A\cos(A-B)$
3. $\tan\left(\frac{\pi}{4}-A\right)=\frac{\cos A-\sin A}{\cos A+\sin A}=\cot\left(\frac{\pi}{4}+A\right)$
4. $\frac{\tan 3A+\tan A}{\tan 3A-\tan A}=2\cos 2A$

H.

1. If $\cos A=-\frac{1}{3}$, find $\sin\frac{1}{2}A$ and $\cos\frac{1}{2}A$.
2. Find sin x, cos x, and tan x, if $\cos 2x=\frac{1}{8}$
3. Prove that $\tan\frac{1}{8}\pi=\sqrt{2}-1$

I. Solve the following equations **without** graphing:

1. $\cos x-2\sin x=1$
2. $3\sin x-5\cos x=4$
3. $6\sin x+3\cos x=\sqrt{5}$

For #4–5 solve in two ways: (i) by expressing cos 2x and sin 2x in terms of t = tan x, and (ii) using an expression of the form $A\sin(2x\pm\alpha)$ or $A\cos(2x\pm\alpha)$.

4. $2\cos 2x+\sin 2x=2$
5. $2\sin 2x-3\cos 2x=\frac{1}{2}$

PART III

A. Evaluate *where possible*. Give exact answers. If a problem requires a calculator in order to solve, then state that a calculator is required, and then evaluate it. Only one problem requires a calculator.

1. $\arcsin\left(-\frac{1}{2}\sqrt{3}\right)$
2. $\arccos\left(\arcsin\frac{3}{5}\right)$
3. $\arctan(\tan(-\pi))$
4. $\cos\left(\arcsin\left(-\frac{1}{2}\right)\right)$

B. 1. For which numbers a is it true that arcsin(sin a) = a?
2. For which numbers b is it true that sin(arcsin b) = b?
3. For which numbers c is it true that arcos(cos c) = c?
4. For which numbers d is it true that cos(arcos d) = d?
In all cases, explain your reasoning.

C. Using the identity $\cos\left(\frac{1}{2}\pi - \theta\right) = \sin\theta$, show that $\arcsin x + \arccos x = \frac{1}{2}\pi$ for all $x \in [-1, 1]$.

D. Find the **exact value** of $\arctan\frac{1}{2} + \arctan\frac{1}{3}$

E. Prove that $2\arctan\frac{1}{3} + \arctan\frac{1}{7} = \frac{1}{4}\pi$

PART IV

A. Differentiate each of the following with respect to x.

1. arcsin(3x)
2. arcos(x + 1)
3. $\arctan\left(x^2\right)$
4. $\ln(\arcsin x)$
5. $x\arctan\left(x^2\right)$

B. For the function $f(x) = \arctan 2x$,

1. Sketch the graph of $y = f(x)$
2. Evaluate $f\left(\frac{1}{2}\right)$
3. Solve the equation $f(x) = 1$
4. Calculate the gradient to the curve at x = 3
5. Find the equation of the tangent line to the curve at $x = -\frac{1}{2}$

PART V

A. Integrate each of the following with respect to x.

1. $\int \frac{3}{\sqrt{9-x^2}}\,dx$

2. $\int \frac{4}{\sqrt{2-3x^2}}\,dx$

3. $\int \frac{6}{9+x^2}\,dx$

4. $\int \frac{2}{3+4x^2}\,dx$

5. $\int \frac{1}{\sqrt{4-(x+1)^2}}\,dx$

6. $\int \frac{1}{4+(x-3)^2}\,dx$

7. $\int \frac{4}{10-2x+x^2}\,dx$

8. $\int_{-5}^{-1} \frac{1}{\sqrt{7-6x-x^2}}\,dx$

B. Integrate each of the following:

1. $\int \frac{2}{\sqrt{1-4x^2}}\,dx$

2. $\int \frac{3x+4}{x^2+4}\,dx$

3. $\int \left(3+\frac{2x}{\sqrt{x^2+4}}\right)^2 dx$

4. $\int_{2}^{2\sqrt{3}} \frac{(1+x)^2}{4+x^2}\,dx$

PART VI

A. Find the area of the segment of a circle of radius 12 cm cut off by a chord of length 10 cm.

B. Two circles of radii 15 cm and 8 cm have their centers 17 cm apart. Find the area common to both circles.

C. A unit circle has a sector cut out of it with a central angle of θ. When a chord is drawn across the sector with the endpoints each touching the circle, the area of the triangle formed is 3 times the area of the remaining area of the sector of the circle. Find θ.